INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - III, Statistics - IV, Final Examination, April 30, 2013

Answer any 5 questions. Maximum possible score is 50.

1. Suppose $D_1, D_2, \dots, D_n$ are continuous random variables which are independent and are symmetric about 0. Let I_j be the indicator variable which is defined as $I_j = 1$ if $D_j \geq 0$ and 0 otherwise, for $1 \leq j \leq n$.
(a) Show that the random vectors $(|D_1|, |D_2|, \dots, |D_n|)$ and $(I_1, I_2, \dots, I_n)$ are independently distributed.
(b) Find the distribution of $\sum_{j=1}^n I_j$. [10]

2. Let $X_1, \dots, X_n$ be a random sample from a continuous distribution with c.d.f. F and density f, both of which are completely unknown. Further, let F_n be the empirical distribution function and let $\{h_n\}$ be a sequence of positive real numbers. Then consider the following estimate f_n of f:

$$f_n(x) = \frac{F_n(x+h_n) - F_n(x-h_n)}{2h_n}.$$

If $h_n = n^{-\alpha}$ with $0 < \alpha < 1$, show that $f_n(x) \longrightarrow f(x)$ in probability as $n \longrightarrow \infty$ at each x. [10]

3. Let $X_1, \dots, X_n$ be a random sample from $N(\theta, \sigma^2)$, where $\theta > 0$ is unknown but σ^2 is known. Consider the decision problem where the loss function is $L(\theta, a) = (\theta - a)^2$, $-\infty < a < \infty$. Show that the decision rule $\delta(X_1, \dots, X_n) = \bar{X}$ is inadmissible as an estimator of θ. (Hint: Consider the decision rule $\delta_1 = \max\{\bar{X}, 0\}$). [10]

4. Solve the 2-person, zero-sum game with the following loss matrix:

	a_1	a_2	a_3	a_4
θ_1	3	3	0	0
θ_2	0	1	3	1
θ_3	0	2	0	2

[10]

5. Consider the 2-person, zero-sum game where player I chooses a number $\theta \in \Theta = [0, 1]$. Player II guesses this to be a number $a \in \mathcal{A} = [0, 1]$. The loss then (to player II) is $L(\theta, a) = \exp(|\theta - a|)$. Find the minimax and maximin strategies. [10]

6. Suppose $X_1, \dots, X_n$ are i.i.d. Bernoulli(θ) where $0 < \theta < 1$ is unknown but n is fixed. Consider estimating θ under the loss
$L(\theta, a) = \theta^{-1}(1-\theta)^{-1}(\theta - a)^2$, $0 \leq a \leq 1$.
Derive the minimax estimator of θ. [10]